<u>Biggleswade Academy Calculation Programme</u>

This document presupposes that you wish to teach calculation with understanding, and not just as a process that is to be remembered. The Calculation Programme clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports the development of mathematical concepts.

The Aims of the curriculum:

The national curriculum for mathematics aims to ensure that all pupils:

Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Recording

Recording is developed in a range of ways, including the following. Although initially they will be developed in this order, once a way of recording, such as 'by showing real objects', is in place, that will <u>continue to be used</u> throughout their schooling, where appropriate. In EYFS most recording will be by showing real objects, whereas in Y6 real objects may be used to show an understanding of calculation with decimals.

Development of recording:

- by showing real objects
- by photographing or drawing the calculation activity
- counting on a number line
- a practical calculation activity on a number line
- a number bond on a number line
- a mental calculation on a number line
- a practical activity as a number sentence
- a number bond as a number sentence
- a mental calculation as a number sentence
- a written calculation

Big Maths

Big Maths is a program that we use across the academy to ensure children develop a strong set of core numeracy skills. It works alongside this document, ensuring that children can relate concepts such as division to different characters that help them enjoy and remember these concepts as they are developed throughout the academy. Currently it is taught from reception to Year 6, although elements of it continue in the teaching at Year 6 and in Years 7 and 8. To help children's recall of key number facts, there are also 36 addition and 36 multiplication 'learn its' (such as 7x8=56) that are practiced from reception to Year 6 to help children know the key number facts to support their core numeracy. You will find references to how Big Maths supports the calculation policy throughout this document. Further information on Big Maths can be found at www.youtube.com/bamaths1.

Progression in calculation

Addition – Early Years / KS1

Children begin calculation purely with practical activities. Over time they learn to record these activities in a way that makes sense to them. This will be by showing or taking photographs of the equipment they have used, leading to drawings of what they did.

For instance, with the practical activity - I have 3 sweets, then I get one more. The child draws the sweets. They may draw 3 sweets and then another. They may just draw 4 to start with.

() () ()   ()

They won't draw 3, then 1, then 4, nor should they be expected to at this stage.

() () ()   () = () () () () doesn't make much sense. You either have 3 and 1 or you have 4. You never have both.

This means that any recording of the format 3+1=4 is very unhelpful and is not based on their experience but on an abstract recording method.

When pupils are ready to record numerals (possibly at the end of the summer term in Year R, but probably in Year 1) they may begin to record the above example with numbers as well:

() () ()   ()

3   **1**   or just as

() () () ()

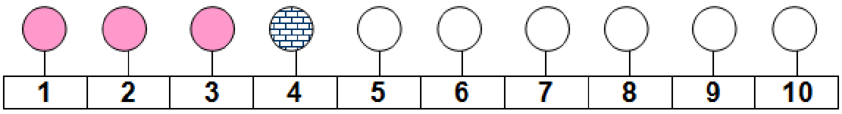
4

but not yet as 3+1, and certainly not as 3+1=4.

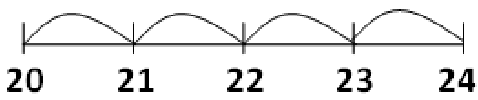
As well as using objects, pupils will begin to use <u>number tracks</u> and then <u>number lines</u> both as practical equipment that makes the calculation transparent and as ways to record what they did. This is also supported by Big Maths through the 'Pim Principle' – if we can count in one thing, then we can count another (so to have 1, 1, 1 apples is the same as having 1, 1, 1 bananas).

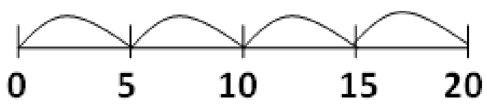
For calculations it is useful to have 'lollipop' number tracks, where counters can be placed in the circles without covering over the numerals.

3 plus 1, for instance:

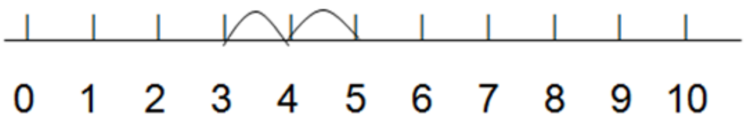


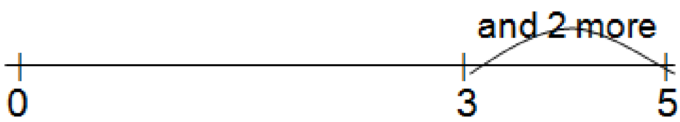
At first children will record their counting on number lines, later moving to recording of calculation on a number line.



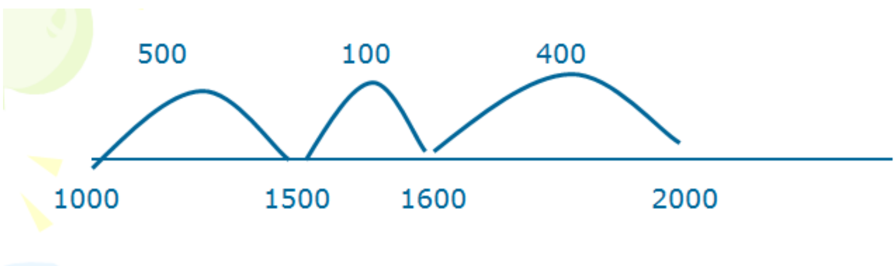


Pupils will use numbered number lines to record jumps, for example for 3+2, before recording on blank number lines.





By the end of Y1 children should be confident using number lines to 'play' with numbers. These examples are from average and above Y1 children at the end of the year. They have counted in steps of 100, 1000, 50 and 500, and have seen their teacher record the numbers they have counted. They believe that maths is about playing with numbers and trying things out rather than just finding the right answer.



Recording number sentences – Key Stage 1 onwards

Before pupils move to recording 3+1 they will need lots of experience of practical addition, and an ability to respond to mathematical vocabulary practically. For instance, if you ask a child to show you 5 and 2 more, or 3 plus 1, or 1 add 4, they can use the teddies, counters or number tracks to show you. They will also be developing their use of mathematical vocabulary to explain what they have done.

From this it will be possible to develop an understanding of the + sign, which will enable pupils to begin to record in the form 5+2.

Pupils then need to understand the concept of equality before using the = sign. This means they can see an example such as 7=6+1, or 5=5, as well as the more common arrangement 3+1=4, and know that it makes sense. This is further supported by the 'Pim Principle' in Big Maths, which states that if you know 3+1=4, then children's understanding of the equals sign can be further deepened by saying that 'if we know this', then 1+3=4 as well. This idea of 'it's nothing new' gives them more contextual understanding of the equals sign and this is then further built on when children start looking at multiplication 'learn its' (facts).

Pupils will still work practically with equipment and real objects, but now can record their explanation of what they have done as a conventional number sentence:

3 + 14 = 17 **17 = 14 + 3** **17 – 3 = 14** **3 + 14 = 14 + 3** and so on.

However, pupils will still record with objects, drawings and number lines **on a frequent basis**, and whenever they are learning new concepts or starting to use a wider range of numbers they will need to return to using these easily understood and explained methods of recording.

Mental methods

Pupils need to develop their use of jottings to support mental calculation. These jottings may be as drawings, number lines or number sentences.

Once children have an understanding of place value in 2-digit numbers, in other words they are convinced that 23 is 20 and 3, or 59 is 50 and 9, they can begin to use partitioning in their mental and recorded calculations.

Partitioning – Year 3 onwards

Partitioning may be recorded in a number of ways, such as:

36 + 45 = 30 + 40 + 6 + 5
= 70 + 11
= 81

536 + 245 = 500 + 200 + 30 + 40 + 6 + 5
= 700 + 70 + 11
= 781

or

36 + 45 = 36 + 40 + 5
= 76 + 5
= 81

The important thing to consider when children are recording partitioning is that they record how they thought about the numbers, and don't all try to do it the same way. This is not about finding lots of ways to record, but of recording what makes sense to a child.

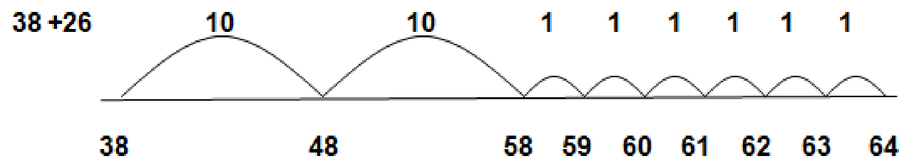
Partitioning is also an appropriate strategy for larger numbers, eventually including decimals.

Partitioning using number lines

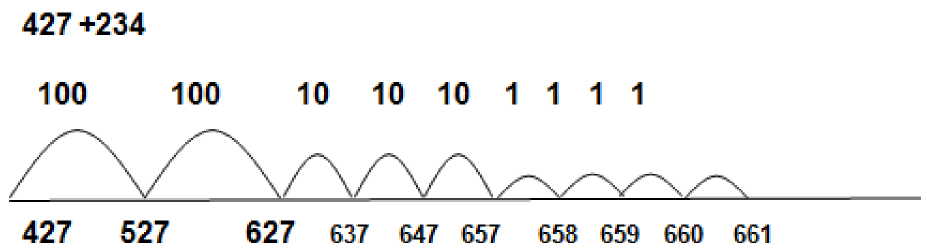
Key understanding – A number line is a tool, not a rule.

Children partition numbers to count on, mainly in multiples of 100, 10 or 1, on a number line. Number lines will be used for calculations right through Key Stage 2, particularly for addition (after all, it's easier to add than subtract!)

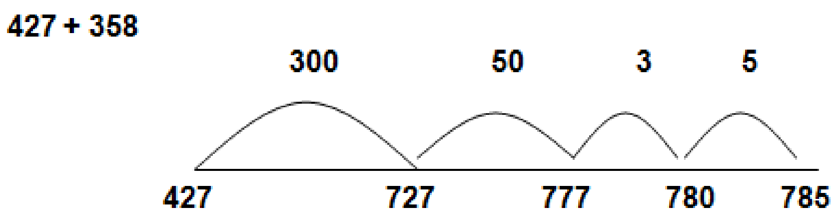
Initial attempts may be a little slow as children choose easy numbers to count on – here are examples that you may see in Key Stage 1:



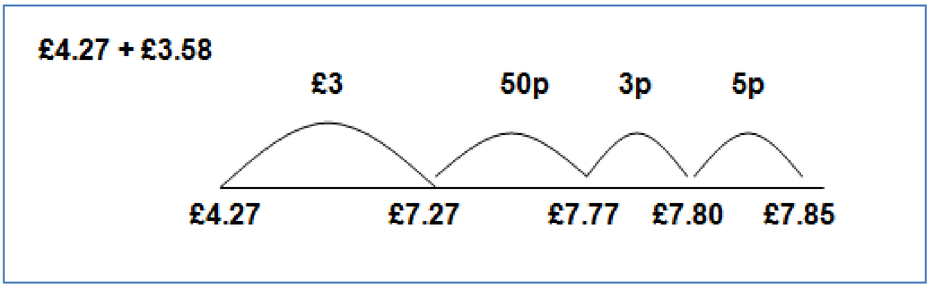
What matters, however, is that children make their own choices of which numbers to use and that they use their understanding of number and place value to find a way that works for them. This may continue into 3-digit numbers for some children.



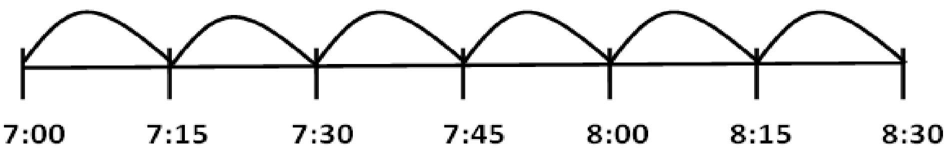
As they become more confident, children start to jump in multiples of 100, 10 and 1. They use their own choice of numbers, doing <u>any</u> jumps on the number line, in steps of 100, 10, 1 or multiples of these, depending on their mental strategies and ability.



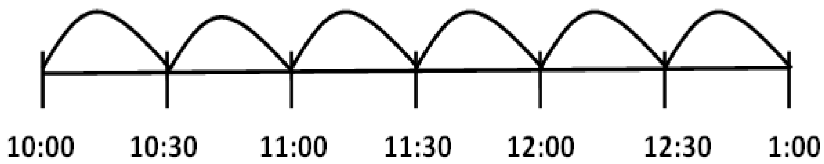
Children need to develop understanding of calculation in a range of contexts, for instance measures, including money and time.

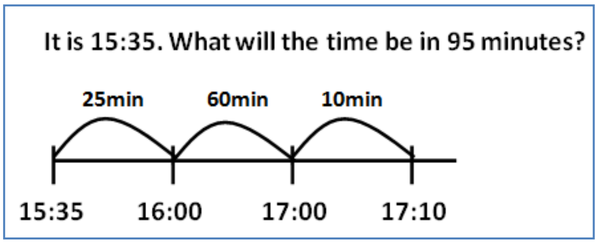


Time is particularly difficult, and at first children will use number lines to record counting in steps of hours or minutes (From Key Stage 1 onwards)



Counting across boundaries is particularly important.





Expanded vertical method

In Year 4 pupils may begin to record addition calculations vertically, at first recording calculations both as the partitioning they have been using and as an expanded vertical calculation, adding numbers in columns, beginning with the hundreds, then tens and then adding the ones. The vocabulary used will always be whole number place value vocabulary, so 254 would be 200, 50 and 4, never 2 hundreds, 5 tens and 4 ones. Always use ‘ones’, as the term ‘units’ is only used for units of measurement and not for place value.
Children discuss what is the same and what is different about each of these ways of recording. They realise that it doesn’t matter in what order you add the totals for the ones, tens or hundreds. The final total is always the same.

$$
\begin{aligned}
547 + 378 &= 500 + 300 + 40 + 70 + 7 + 8 \\
&= 800 + 110 + 15 \\
&= 925
\end{aligned}
$$

	5	4	7
+	3	7	8
	8	0	0
	1	1	0
		1	5
	9	2	5

	5	4	7
+	3	7	8
		1	5
	1	1	0
	8	0	0
	9	2	5

The expanded vertical method also works alongside the use of number lines to find the answer in subtraction questions for Years 3 and 4. It may continue to be useful for children who find adding larger numbers difficult beyond, as its links to partitioning are much clearer than the method of column addition.

From Year 6 onwards, children will begin to be introduced to column addition, where appropriate. This method works quickly, but requires a secure understanding of the bridging of numbers between hundreds, tens and ones. In the example below, you would add from right to left, so 6+9=15. You write the 5 in the ‘ones’ column and carry the ten over into the tens column. In the tens column this now has the same relative value as the other numbers in the column, so in the tens column you do 5+3+1=9. As there is no carry over, the final column is more straight-forward being 4+2=6, giving a final answer of 695. **It is expected that the expanded vertical method is still used and accepted, particularly with children who find adding difficult.**

	H	T	O
	4	5	6
+	2	3	9

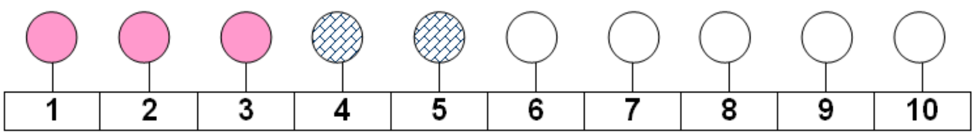
	H	T	O
	4	5	6
+	2	3	9
		1	
			5

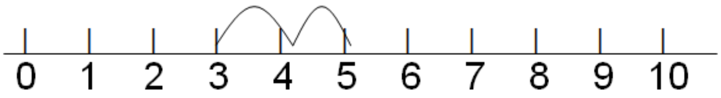
	H	T	O
	4	5	6
+	2	3	9
		1	
	6	9	5

Subtraction – Early Years

As with addition, subtraction is initially recorded as drawing the result of a practical activity, moving on to record this using numbers, on number tracks or lines or as number sentences.

Initially <u>number tracks or lines</u> will be used to subtract small numbers such as 5 – 2.





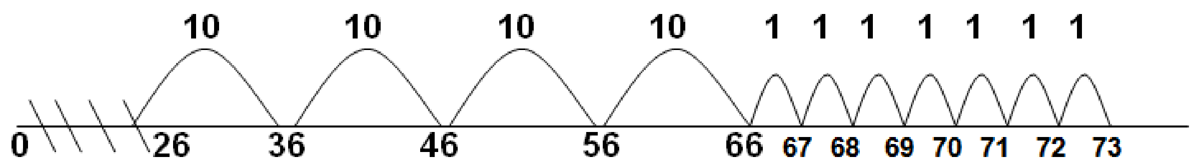
When pupils move on to use jottings (more Year 1 and 2) the number line will become especially important. Jottings as number sentences are less useful for subtraction as partitioning cannot generally be used.

In the example 73 – 26 it is possible to start with 70 – 20, but 3 – 6 is less useful!

Key understanding – Pupils need to realise that partitioning is not appropriate for subtraction.

Number lines, however, make calculating easier – here's an example you might see in KS1:

73 – 26 =

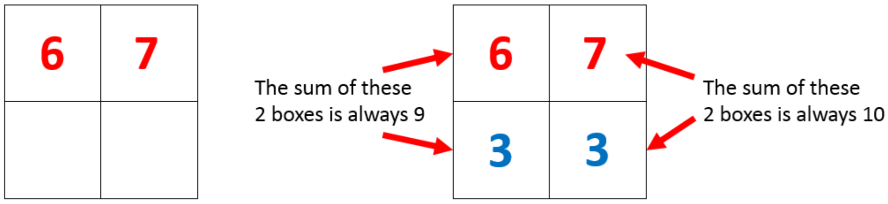


Key understanding – Putting the zero on a number line for subtraction and crossing out what has been subtracted makes the subtraction obvious.

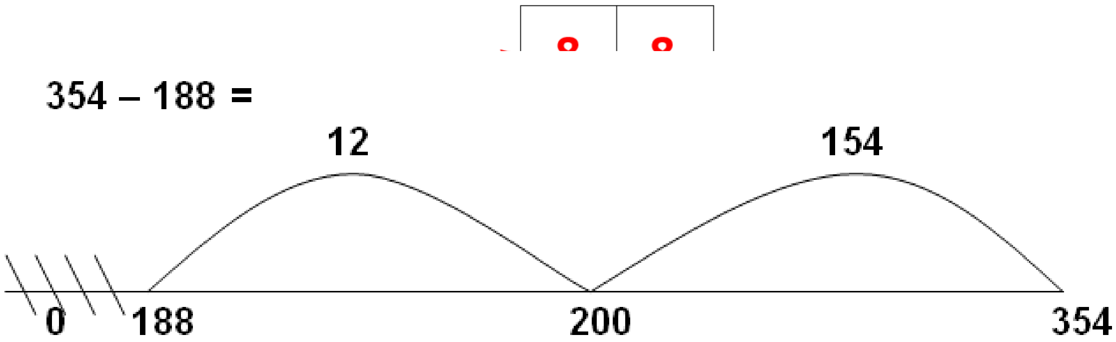
You'll notice that there is a zero placed on the number line. This helps to stop children writing the 73 on the left hand side of the number line, but more importantly enables you to cross out and 'take away' the 26. It makes it easier to understand that this is a subtraction, and you are counting on to find out how many are left.

You will have noticed, (a bit like in the first steps of using a number line for addition), that the first number lines that use subtraction contain a number of jumps – this is fine as the children are doing something that they feel comfortable with. However, as their understanding increases we are able to talk about and build an understanding of an 'efficient' subtraction number line – the fewer jumps there are, the better chance the children have of getting the right answer (a common mistake on the number line above is that, despite the number line itself being correct, the children get confused adding all the tens and ones together and present an incorrect final answer).

To develop their understanding of an efficient number line, we use **Jigsaw Numbers** from Big Maths from Year 1 onwards (although it is not explicitly linked to subtraction until upper Key Stage 2). Jigsaw numbers thinks about how we find numbers that bond to first ten and then a hundred. So for 67 we would get the children to do this:



So, not only does this help the children think about bonds to 10 and 100 (key mental strategies for addition and subtraction), it becomes useful when doing 3 digit numbers subtracted, such as 354-188:

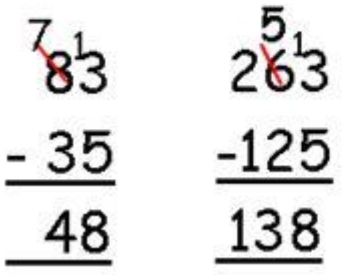


This is a much more effective number line because there are only 2 numbers to add at the end to find the final answer (12+154=166). We get the children to find the next 100, rather than finding the nearest ten, which sounds counterproductive, until you ask them to use their knowledge of Jigsaw Maths, which you can see on the number line above, so if they find the to the nearest hundred, then it's just 12 to 200. When you're on the nearest hundred, it's then easier to do the next jump to the final total (for a child who wasn't comfortable finding this, a jump of +100 & +54 would still be appropriate).

We also continue to use number lines for subtraction calculations in a range of contexts, such as time, money, mass, length and capacity. With time (as in the addition examples) there is a need to be careful and recognise that this will be different.

The use of the column subtraction method

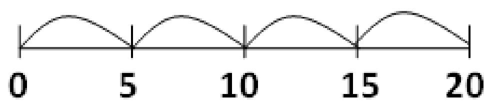
In some cases we may ask children who are particularly confident in maths to experiment with this method, but as a general rule, it can often lead to confusion, as the children don't really understand the idea of borrowing and editing, so as a general rule this method is not taught explicitly in Key Stage 2. Children in Key Stage 3 however may begin to use it, along with those who have shown a confidence in using a variety of methods so far.



An example of column subtraction **(for reference, this is not taught explicitly in lessons; children are expected to do subtraction calculations on a number line)**

Multiplication – Early Years

Children's first recording in multiplication will be by placing objects in arrays and counting in steps on number lines from zero. This is supported in Big Maths by the children counting along these number lines as a whole class.



Concepts of multiplication develop using doubling and counting in steps, and are extended using the array. Objects, arrays, number lines and number sentences will continue to be the main methods of recording.

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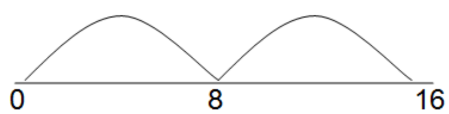
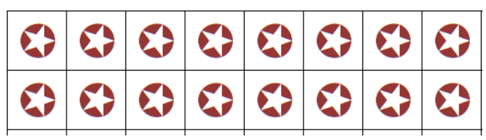
8 x 2 = 16

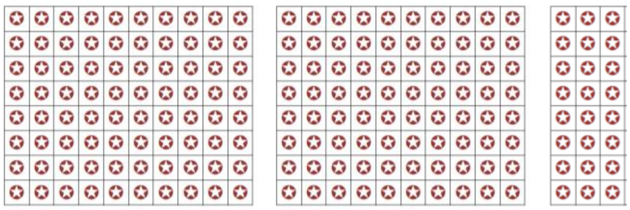




8x2 means I start from zero and count on 8 twice.

Once pupils begin to multiply one-digit by two-digit numbers this will be by using partitioning (From end of Key Stage 1 / Start of Key Stage 2). Pupils will be unlikely to have used brackets at this stage and it is best to let them record without brackets, but with a clear understanding of what they are doing, based on an understanding of arrays and a diagram to explain the calculation.

8 x 23



8 x 23 = **8 x 10** **+** **8 x 10** **+ 8 x 3**

= 80 + 80 + 24

= 184

This leads to the grid method of multiplication:

X	**10**	**10**	**3**	
8	80	80	24	=184

Once children can show an understanding of a 1-digit by 2-digit multiplication both with an array and a grid multiplication they can explore multiplying a multiple of 10 by a 1-digit number. Using this decreases the number of steps needed to complete the multiplication.

X	**20**	**3**	
8	160	24	= 184

The grid method can then be used for 2-digit by 2-digit multiplication. At first just use numbers between 11 and 19. For instance try 16 x 14:

X	**10**	**6**	
10	100	60	
4	40	24	
	= 140	= 84	= 224

Using numbers 11 to 19 keeps the mental calculations relatively simple.
When adding together the four totals, this can be done either horizontally or vertically.

Later children can move on to other 2-digit numbers and decimals.

X	10	6	
10	100	60	=160
4	40	24	= 64
			= 224

66 x 34

X	60	6	
30	1800	180	
4	240	24	
	= 2040	= 204	= 2244

73.5 x 17

X	70	3	0.5	
10	700	30	5	
7	490	21	3.5	
	= 1190	= 51	= 8.5	

Addition of the products may become a separate addition calculation.

					.	
	1	1	9	0	.	0
			5	1	.	0
+				8	.	5
	1	0	0	0	.	0
		1	0	0	.	0
		1	4	0	.	0
				9	.	0
	1	2	4	9	.	5

					.	
	1	1	9	0	.	0
			5	1	.	0
+				8	.	5
	1	2	4	9	.	5
		1				

Eventually, towards the end of Year 5 and certainly through Year 6 onwards, we move children on to the Column Multiplication method, although this will not be explicitly taught until September 2015, as it will form part of the methods used in 2016 SATs paper.

Division – from Key Stage 1

As with multiplication, division is recorded with objects, arrays, number lines or number sentences. Through the character of Mully in Big Maths, children are introduced to the concept of division, but a bit like with using a number line and addition to solve subtraction problems, we make use of the children's developing strength and confidence in using multiplication to solve division problems.

So, with 'Where's Mully?' we ask the children – where's Mully hiding?

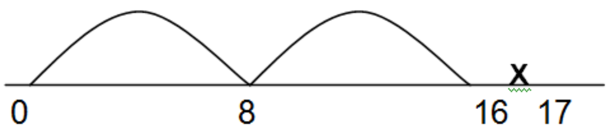
Mully is hiding behind the biggest multiple of….

Without going past…

39! So, the answer here gets the children to use their counting skills and knowledge of multiplication through the 3x table…Eventually (in Key Stage 2), you might start linking Where's Mully more explicitly with division questions, such as the one below.

As we start, we may use number lines to record this process – this can also help with introducing the concept of remainders.

So, 17 ÷ 8 = 2 with 1 left over



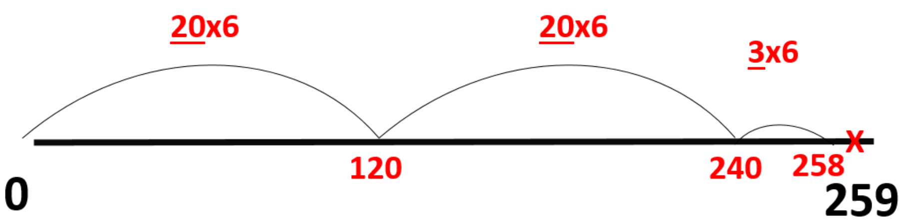
As the children progress into Year 5, the division questions will start to involve larger numbers, outside the range of the children's times table knowledge. For that, we begin developing the formal method of **long division.**

For many pupils, the addition 'Coin Multiplication' from Big Maths makes the calculation easier for the child. It's called Coin Multiplication because it gets the children to think of all the coins from 1p to £1 (or 100p) and then think of them as multiples, so a completed coin card looks like this:

For 259 ÷ 6:

X6	
X1	**6**
X2	**12**
X5	**30**
X10	**60**
X20	**120**
X50	**300**
X100	**600**

So in this example, you would get the child to think of playing ‘Where’s Mully?’ just with a much bigger number – so Mully is hiding behind a multiple of 6 without going past 259 – so where is he? The number line would therefore look like this:



Looking at my coin card I could see that I could get 300 by doing 50x6 but this would take me *past* 259, and Where’s Mully has taught the children not to go past that. So instead you look to the multiple below it – in this case 20x6=120. I now put this on my number line and see that if I did it again, I would get to 240 – much closer to my total, still without going past it. The multiples of 6 are underlined because this is the part that I will add together at the end to get to my answer. So, I eventually get to 258 and can’t go any closer without going past – 1 remains to get to 259, so my answer is 20+20+3 with a reminder of 1, so 43 r1.

At the end of Key Stage 2 we would encourage children to not use a number line and complete the formal written steps of long division, but we would also expect that some children may need to continue using a number line to support their ability to solve these complex division questions.

259 ÷ 6

	6	**259**
(40x6)		**240**
		19
(3x6)		**18**
		1

	X6
X1	**6**
X2	**12**
X5	**30**
X10	**60**
X20	**120**
X50	**300**
x100	**600**

Appendix

Examples of written methods for addition, subtraction, multiplication and division, suitable for the end of Key Stage 2, going into Key Stage 3.

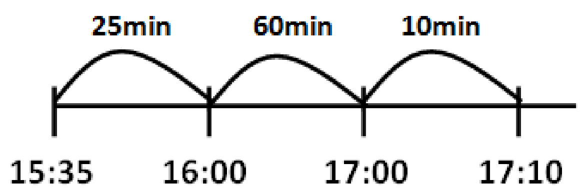
These examples can be taught with understanding rather than as remembered processes only.

Addition

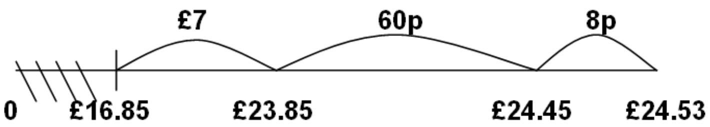
5	3	.	2
	4	.	9
5	0	.	0
	7	.	0
	1	.	1
5	8	.	1

2	7	5	3	.	2
3	6	5	4	.	9
6	4	0	8	.	1
1	1		1		

It is 15:35. What will the time be in 95 minutes?



Subtraction



£24.53 - £16.85 = £7.68

	200	100+40	10+4
	~~300~~	~~50~~	~~4~~
-	100	80	8
	100	60	6

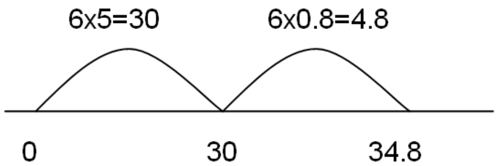
Multiplication

X	70	3	0.5	
10	700	30	5	
7	490	21	3.5	
	= 1190	= 51	= 8.5	

X	100	70	3	
20	2000	1400	60	= 3460
7	700	490	21	= 1211
				= 4671

		1	7	3	
x			2	7	
2	*0*	*0*	*0*		20x100
1	*4*	*0*	*0*		20x70
		6	*0*		20x3
	7	*0*	*0*		7x100
	4	*9*	*0*		7x70
		2	*1*		7x3
4	6	7	1		

Division



34.8 ÷ 6 = 5.8

		1	2	4	r2
14	1	7	3	8	
	1	4	0	0	100x14
		3	3	8	
		2	8	0	20x14
			5	8	
			5	6	4x14
				2	